## Homework 1 (Observations)

You may make any reasonable assumption or use numerical information from the internet to solve these problems. Mention your assumption or borrowed information if you do.

**1. To Build the Event Horizon Telescope:**
If the accretion flow around a black hole is observed with enough angular resolution it would look like a dark shadow surrounded by a bright ring. The radius of the inner edge of this ring is approximately 5.2 $r_g$, where $r_g = GM_{BH}/c^2$. M87 is the nearest galaxy with an actively accreting super-massive black hole at the center. Its distance is 16.8 Mpc and the central black hole's mass is $6 \times 10^9$ $M_{sun}$.
a) What is the angular resolution needed to resolve the ring described above for M87?
b) What is the angular resolution of the Keck telescope, the largest optical telescope on Earth with a diamater of 10m?
c) If the required resolution is achieved by radio interferometry at a wavelength of 10 cm, how long should the baseline be? Is that possible on Earth's surface?
d) What wavelength may be used for interferometry if the maximum baseline that can be created is 10,000 km?

**2. Dimming of Betelgeuse with Naked-Eye Observation:**
a) Betelgeuse is the bright reddish star on the left hand of the Orion constellation. During 2019 Dec-2020 January it was exhibiting an anomalous behavior. It was becoming continuously fainter. In the last two months its magnitude has increased by 0.5. What should be the sensitivity of our eye for us to detect this change by naked-eye observation? In other words how small a fractional change we must be able to detect in order to detect Betelgeuse's dimming?
b) Consider the $V-$band in which the standard star Vega's magnitude is zero and Betelgeuse's magnitude is 1.0. The $V-$band is centered at 550 nm and has a width of 100 nm. Vega's flux in $V-$band is $3.5 \times 10^{-8}$ erg cm$^{-2}$ $s^{-1}$ $nm^{-1}$. How many $V-$band photons per second do we detect by our eyes from Betelgeuse before and after its dimming?

**3. Optional Problem—The diffraction limit of a telescope:**

Our ability to resolve detail in astronomical observations is limited by the diffraction of light in our telescopes. Light from stars can be treated effectively as coming from a point source at infinity. When such light, with wavelength $\lambda$, passes through the circular aperture of a telescope (which we'll assume to have unit radius) and is focused by the telescope in the focal plane, it produces not a single dot, but a circular diffraction pattern consisting of central spot surrounded by a series of concentric rings. The intensity of the light in this diffraction pattern is given by

$$I(r) = \left(\frac{J_1(kr)}{kr}\right)^2,$$

where $r$ is the distance in the focal plane from the center of the diffraction pattern, $k = 2\pi/\lambda$, and $J_1(x)$ is a Bessel function. The Bessel functions $J_m(x)$ are given by

$$J_m(x) = \frac{1}{\pi}\int_0^{\pi} \cos(m\theta - x\sin\theta)\,\mathrm{d}\theta,$$

where $m$ is a nonnegative integer and $x \geq 0$.

1. Write a function `J(m,x)` that calculates the value of $J_m(x)$ using Simpson's rule with $N = 1000$

points. Use your function in a program to make a plot, on a single graph, of the Bessel functions $J_0$, $J_1$, and $J_2$ as a function of $x$ from $x = 0$ to $x = 20$.

2. Make a second program that makes a density plot of the intensity of the circular diffraction pattern of a point light source with $\lambda = 500\,\text{nm}$, in a square region of the focal plane, using the formula given above. Your picture should cover values of $r$ from zero up to about $1\,\mu\text{m}$.

Hint 1: You may find it useful to know that $\lim_{x\to 0} J_1(x)/x = \frac{1}{2}$. Hint 2 (if you are using Python): The central spot in the diffraction pattern is so bright that it may be difficult to see the rings around it on the computer screen. If you run into this problem a simple way to deal with it is to use one of the other color schemes for density plots described in Section 3.3. The "`hot`" scheme works well. For a more sophisticated solution to the problem, the `imshow` function has an additional argument `vmax` that allows you to set the value that corresponds to the brightest point in the plot. For instance, if you say "`imshow(x,vmax=0.1)`", then elements in `x` with value 0.1, or any greater value, will produce the brightest (most positive) color on the screen. By lowering the `vmax` value, you can reduce the total range of values between the minimum and maximum brightness, and hence increase the sensitivity of the plot, making subtle details visible. (There is also a `vmin` argument that can be used to set the value that corresponds to the dimmest (most negative) color.) For this exercise a value of `vmax=0.01` appears to work well.