Homework 2 (Stellar Parameters and Spectra)

Exercise 2.1: Stellar Mass Measurement from Spectroscopic Binary Systems:
The maximum radial velocities measured for the two components of a spectroscopic binary are 100 and 200 km s^{-1}, with an orbital period of 2 days.
i) What is the value of $Msin^3 i$ for each star, where M is the mass and i is the inclination to the observer's line of sight of the perpendicular to the orbital plane.
ii) Find the mean expectation value of the factor $\sin^3 i$, i.e., the mean value it would have among an ensemble of binaries with random inclinations.
iii) Assuming the distribution of i to be random is not right. Is it? Comment.

Exercise 2.2: Stellar Parameters from an Eclipsing Spectroscopic Binary:
In an eclipsing spectroscopic binary, the maximal radial velocities measured for the two components are 20 and 5 km s^{-1}. The orbit is circular, and the orbital period is P=5 yr. It takes 0.3 day from the start of the eclipse to the main minimum, which then lasts for 1 day.
a) Find the mass of each star. Since the binary is of eclipsing type, one can safely asume i $\simeq 90°$. Check to what degree the results are affected by small deviations from this angle, to convince yourself that this is a good approximation. (b) Assume again i = 90° and find the radius of each star. Is the result still insensitive to the exact value of i?

Exercise 2.3: Relative Strengths of Absorption Lines:
The temperature of the solar photosphere is 5777 K and it has 500,000 hydrogen atoms for each Ca atom. The electron pressure is 1.5 N m^{-2}, Z_I = 2, Z_{II} = 1 for H and Z_I = 1.32, Z_{II} = 2.3 for Ca. Estimate the relative strengths of the absorption lines due to hydrogen Balmer lines and those due to Ca II H and K lines (produced by electrons in the ground state of singly ionized Ca).

Exercise 2.4: Numerical Simulation of Exoplanet Orbital Orientation
This is an extra problem. You do not need to submit this. Suppose you wish to choose a random point on the surface of the Earth. That is, you want to choose a value of the latitude and longitude such that every point on the planet is equally likely to be chosen. *In a physics context, this is equivalent to choosing a random vector direction in three-dimensional space.*

Recall that in spherical coordinates θ, ϕ the element of solid angle is $\sin\theta \, \mathrm{d}\theta \, \mathrm{d}\phi$, and the total solid angle in a whole sphere is 4π. Hence the probability of our point falling in a particular element is

$$p(\theta, \phi) \, \mathrm{d}\theta \, \mathrm{d}\phi = \frac{\sin\theta \, \mathrm{d}\theta \, \mathrm{d}\phi}{4\pi}.$$

We can break this up into its θ part and its ϕ part thus:

$$p(\theta, \phi) \, \mathrm{d}\theta \, \mathrm{d}\phi = \frac{\sin\theta \, \mathrm{d}\theta}{2} \times \frac{\mathrm{d}\phi}{2\pi} = p(\theta) \, \mathrm{d}\theta \times p(\phi) \, \mathrm{d}\phi.$$

1. What are the ranges of the variables θ and ϕ? Verify that the two distributions $p(\theta)$ and $p(\phi)$ are correctly normalized—they integrate to 1 over the appropriate ranges.

2. Find formulas for generating angles θ and ϕ drawn from the distributions $p(\theta)$ and $p(\phi)$. (The ϕ one is trivial, but the θ one is not.)
3. Write a program that generates a random θ and ϕ using the formulas you worked out.
4. Modify your program to generate 500 such random points, convert the angles to x, y, z coordinates assuming the radius of the globe is 1, and then visualize the points in three-dimensional space using any software.
5. Can you use this program to verify the results you obtained above regarding the average value of $sin^3 i$, where i is the inclination of the orbit of exoplanets?