Problem Set 3: Conservation of Energy

1. **Collision in Two Different Frames:**
A mass M, initially moving at speed v, collides and sticks to a mass m, initially at rest. What are the final energies of the two masses, and how much energy is lost, in:
(a) The lab frame? (b) The frame in which M is initially at rest?
Assume M >> m which means M/(M+m) may be approximated as (1-m/M) and higher order terms may be neglected.

2. **Equation of Motion from Energy:**
Here is another example of how the consideration of energy can lead to the equation of motion without the use of F=ma. Write down the total energy E of the bob of a simple plane pendulum. Now differentiate E with respect to time to find the equation of motion.

3. **Jumping Balls:**
A small ball of mass m is placed on top of another ball of mass M, and the two balls are dropped to the floor from height h. How high does the small ball rise after the collision? Assume the collision with the larger ball is elastic, $m << M$, and the balls are slightly separated when the larger ball hits the floor.

4. **Small Oscillation:**
A particle of mass m moves in one dimension along the positive x axis. It is acted on by a constant force directed toward the origin with magnitude B, and an inverse-square law repulsive force with magnitude A/x^2.
(a) Find the potential energy function $U(x)$.
(b) Sketch the energy diagram for the system when the maximum kinetic energy is $K_0 = \frac{1}{2}mv_0^2$.
(c) Find the equilibrium position, x_0.

5. **Bouncing Ball between Moving walls (This is an extra problem. You do not have to submit this):**
Our Universe is able to produce high energy particles very regularly having energy much higher than those produced in the best particle accelerators scientists have been able to design, e.g., Large Hadron Collider at CERN which accelerates protons to 1 TeV. In contrast, cosmic rays can have energy as high as 10^{20} eV, which is equivalent to a subatomic particle containing the KE of a tennis ball thrown at 70 km h^{-1}. It is a mystery how such acceleration occurs. Enrico Fermi showed that charged particles bouncing back and forth between moving magnetic fields in interstellar or intergalactic medium may be a conceivable process that can produce such acceleration. In this problem we shall verify this idea with an example from classical mechanics.

A ball of mass m bounces back and forth elastically with speed v_0 between two surfaces kept

at a distance l. Neglect gravity.
a) If one surface is slowly moved toward the other with speed V $<<$ v, the bounce rate will increase due to the shorter distance between collisions, and because the ball's speed increases when it bounces from the moving surface. Find the average force on each surface as a function of the separation x between them.
b) Show that the work needed to push the surface is equal to the gain in KE of the ball.
(*This is also related to how a gas heats up when compressed.*)