Homework 5: Pulsars and Close Binary Systems

1. **Pulsar Energy Source:**
Pulsar's loss of rotational energy is one hypothesised source of its energy. Luminosity of the Crab nebula is 5×10^{38} erg s^{-1}, its angular frequency (ω) is 190 s^{-1} and the rate of change of its angular frequency (dω/dt) is 2.4×10^{-9} sec^{-2}. If the above hypothesis is true what is its moment of inertia? From what you know about the moment of inertia of a solid sphere, is the value you found plausible for a neutron star?

2. **Runway Fusion in Degenerate Material:**
Explain briefly the similarity of *He flash* during stellar evolution and *nova* in WD binaries. In each case explain what stops the runaway fusion process. Otherwise all He flash and novae would become a large stellar explosion. Explain what is different in the case of Type 1A supernovae, which actually is a large stellar explosion.

3. **The Eddington Weight-Loss Program**
— "Are you serious!" the man said with a horrified face.
— "Yes sir," the employee seemed unperturbed.
— "Losing the weight was hard enough and now I have to jump towards a black hole?"
— "You see sir, this has been the novelty of our program all along," The employee patiently explained. "Otherwise why would anyone choose us over the ones promoted by the movie stars?"
— "But are you sure I shall not be swallowed by the black hole?"
— "Not if you have lost enough weight. That is our challenge."
— "But what stops me from going in? My weight is not zero, you know?" The man is still not convinced.
— "Radiation pressure, sir."
— "What do you mean?"
— "Well, the radiation from the central object is equal to the Eddington luminosity corresponding to the mass of the black hole. Therefore, your weight may be supported by the radiation pressure when you jump from our ship."
— "May be? My life hinges on a *may be*?"
— "Well, that is why the special suit, sir. We make the suit with an appropriate cross-sectional area. We even remotely change the color of your suit from black to white. Less we have to turn the knob to white more weight you have lost."
— "What if I have not lost any weight?"
— "That's impossible, sir. Our program is excellent. At least we are sure you have not gained any weight while in our program. That is enough to save you at the highest level of whiteness."
— "But what is the point of all this?"
— "You see sir, when you jump off the ship and float in the light, you never forget that feeling of

... err ... lightness. Some say it's "unbearable."
— "So?"
— "You never go back, sir. You never gain the lost weight. You are always conscious. You never want to lose that feeling. That's why we give the lifetime warranty. The movie stars can never give that."

If the man started this futuristic weight loss program with an initial weight of 100 kg what should be the cross-sectional area of his suit designed by the company?

4. **Eating Habits of Black Holes:**
a) Show that for a mass, m, the change in potential energy when it is brought from an infinite distance up to the Schwarzschild radius of a black hole is equal to half the rest-mass energy.

b) Calculate the amount of mass that a black hole must "eat" in order to sustain an energy output of 10^{12} L_{sun}, where the source of energy is solely the conversion of potential energy. Assume the efficiency of conversion of potential energy is $\sim$10%.

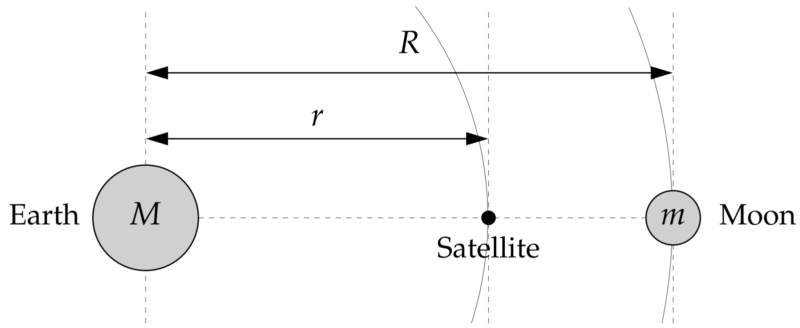
c) The closest distance, d_R, that a star with mean density ρ_* can come to another body with mean density ρ and radius R without being tidally disrupted is the so called "Roche limit":

$$d_R = 2.44 \left(\frac{\rho}{\rho_*}\right)^{1/3} R$$

Calculate d_R for a solar type star and black holes with mass of 10^8 and 10^9 M_{sun}. If these black holes were to dine on stars instead of gas disks, would they bother to chew their food or would they simply swallow them whole?

5. **The Lagrange point (This is a numerical problem. You do not have to submit this):**

There is a magical point between the Earth and the Moon, called the L_1 Lagrange point, at which a satellite will orbit the Earth in perfect synchrony with the Moon, staying always in between the two. This works because the inward pull of the Earth and the outward pull of the Moon combine to create exactly the needed centripetal force that keeps the satellite in its orbit. Here's the setup:



1. Assuming circular orbits, and assuming that the Earth is much more massive than either the Moon or the satellite, show that the distance r from the center of the Earth to the L_1 point satisfies
$$\frac{GM}{r^2} - \frac{Gm}{(R-r)^2} = \omega^2 r,$$
where M and m are the Earth and Moon masses, G is Newton's gravitational constant, and ω is the angular velocity of both the Moon and the satellite.

2. The equation above is a fifth-order polynomial equation in r (also called a quintic equation). Such equations cannot be solved exactly in closed form, but it's straightforward to solve them numerically. Write a program that uses either Newton-Raphson or any other method of your choice to solve for the distance r from the Earth to the L_1 point. Compute a solution accurate to at least four significant figures.

   The values of the various parameters are:
$$\begin{aligned} G &= 6.674 \times 10^{-11}\,\mathrm{m^3 kg^{-1} s^{-2}}, \\ M &= 5.974 \times 10^{24}\,\mathrm{kg}, \\ m &= 7.348 \times 10^{22}\,\mathrm{kg}, \\ R &= 3.844 \times 10^{8}\,\mathrm{m}, \\ \omega &= 2.662 \times 10^{-6}\,\mathrm{s^{-1}}. \end{aligned}$$

   You will also need to choose a suitable starting value for r, or two starting values if you use the secant method.