Homework 6: Galaxies

1. **Disk-to-Bulge:**
For a "disk plus bulge" model of a spiral galaxy, the surface brightness of the disk is modeled as:

$$\Sigma(r) = \Sigma_0 exp(-r/r_s),$$

where r_s is a constant, and Σ_0 is the surface brightness at the center. On other hand, the surface brightness of the bulge is modeled as the so called "de Vaucoleurs law" :

$$\Sigma(r) = \Sigma_e exp(-7.67[(r/r_e)^{\frac{1}{4}} - 1]),$$

where r_e is a constant called the "half-light radius," and Σ_e is the surface brightness at r_e. The units of $\Sigma(r)$ in both the cases are solar luminosities per square parsec. Using this information compute the ratio of the total disk luminosity D to the total bulge luminosity B. D/B is a key parameter used to describe disk galaxies.

2. **Spherical Galaxy:**
Consider a spherical galaxy whose gravitational potential is given by

$$\phi(r) = -\frac{GM}{\sqrt{r^2 + a^2}},$$

where a is a constant.

a) Compute the mass of the galaxy interior to a radius r. Demonstrate that M is the total mass of the galaxy (i.e., at $r \to \infty$). **Hint:** Think about Poisson's equation.

b) Compute the total potential energy of the galaxy. **Hint:** Think about self-gravitational potential energy.

3. Explain briefly how it is known that AGN have a super-massive black hole at their center?

4. **20th Century Astronomy (You do NOT need to submit this problem):**
Imagine that you are a pre-20th century astronomer who is contemplating the night sky. You begin by imagining that we live in a Universe which is both infinite and non-evolving. You also imagine that the Universe consists only of stars similar to the Sun and that the stars are uniformly distributed throughout space with density n_0.

a) For a thin spherical shell of radius r and thickness dr, calculate the number of stars you would expect to find on average in such a shell and hence, the flux contributed by each shell.

b) Divide the hypothetical Universe up into concentric spherical shells and treat all stars as if they were points, rather than spheres of finite radius. What would you measure as the total flux for the Universe? In this scenario, would the night sky be dark or bright?

c) Stars are, of course, not points and as a result the answer to part (b) is non-sensical. Why? If all the stars are spheres of radius R_{sun}, what would you expect the total flux to be? You don't have to do a calculation here. Just draw a picture of what the sky would look like and use it to think about what the answers would be. What would be the color of the night sky?

d) The answer to part (c) caused some pre-20th century astronomers to postulate that the Universe must have an "edge." Why?

e) Imagine that the stars are embedded in a gas which is uniformly distributed throughout the Universe. Without performing any calculations, would the night sky be dark in such a case? Why or why not? What if instead of a gas, dust was spread uniformly throughout the Universe?

f) Based solely up on our present-day understanding of the stellar distribution within our own Galaxy, why should the night sky be dark?

g) Replace the stars in the infinite, non-evolving Universe above by galaxies, all of which have some characteristic luminosity, L_*. Qualitatively, what would you expect the night sky to look like?

h) Imagine the Universe is infinite in extent and the galaxies are distributed throughout it uniformly. Now argue in favor of the existence of a dark night sky based on a finite age of the Universe and an appeal to 20th century physics.

This should motivate you to start studying modern cosmology.